convex risk

Equilibrium Risk Pools in a Regulated Market with Costly Capital

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October 27, 2020

Why is Florida homeowners written in monoline companies?

Context and Literature

- Capital allocation and multiline pricing: perfect markets with frictional costs of holding capital and ex post **equal priority** default rule
  - Phillips, Cummins, Allen (JRI 1998)
  - Myers, Read (JRI 2001)
  - Sherris (JRI 2006)
  - Ibragimov, Jaffee, Walden (JRI 2010)
  - Cummins (RMIR 2000): frictions caused by tax, regulation and agency problems

- We assume the opposite: **imperfect market** but **no frictional costs of capital**
  - Risk cost of capital is not a friction
  - Rationale: catastrophe bond pricing

Context and Literature

- Charge for risk using a non-additive **distortion (spectral) risk measure** (DRM)
  – Wang (ASTIN 1996), Wang, Young, Panjer (IME 1997)

- Possible rationale: **ambiguity averse** investors charge for shape of risk
  – Klibanoff, Marinacci & Mukerji (Econometrica 2005)

- DRMs are non-additive, but they are still **consistent with general equilibrium** and **no arbitrage** prices
  – De Waegenaere, Kast, and Lapied (IME 2003), Chateauneuf, Kast, Lapied (Math Fin 1996)

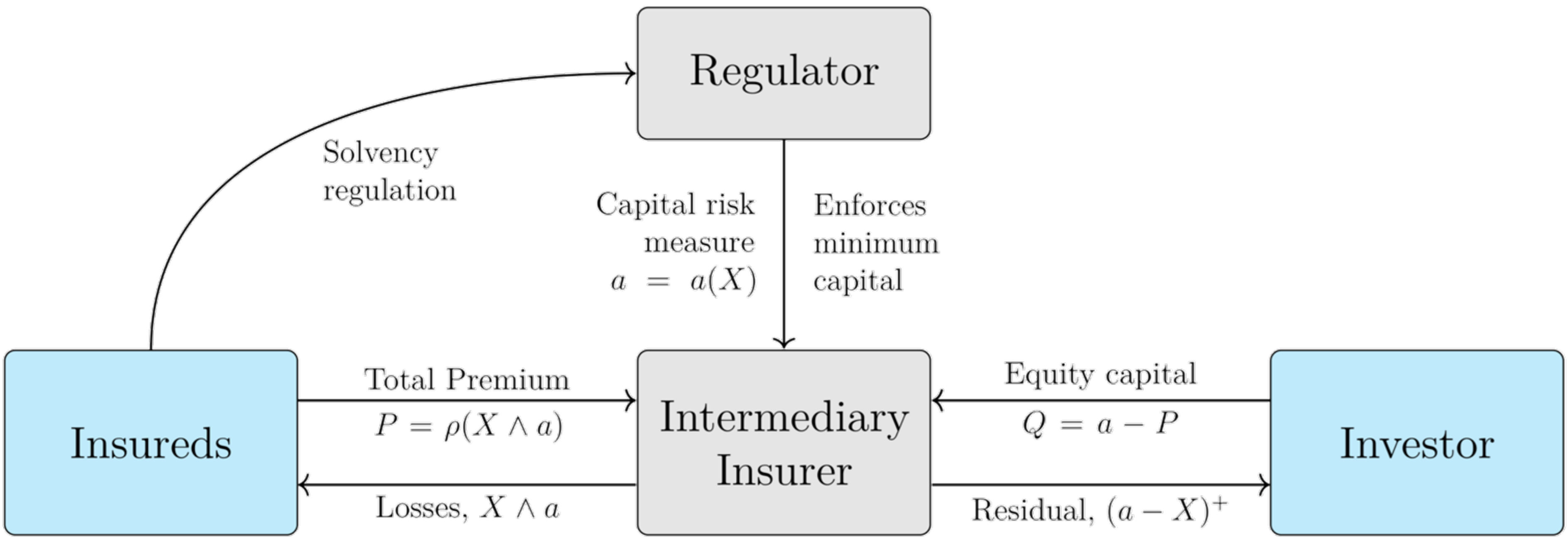
Context and Literature

- Diversification traps: Ibragimov, Walden (JB&F 2007) applies with very thick tails

- Ibragimov, Jaffee, Walden (Rev Fin 2018)
  - Perfect market with frictional cost of holding capital
  - One-sided protection rather than risk pooling
  - "***Basic structure questions in a risk market with one-sided protection remain unanswered.***"
  - Show monoline solutions more likely when risks asymmetric or correlated
  - We show qualitatively similar results with entirely different assumption

- Presentation partly based on joint work with John Major (arxiv 2020)

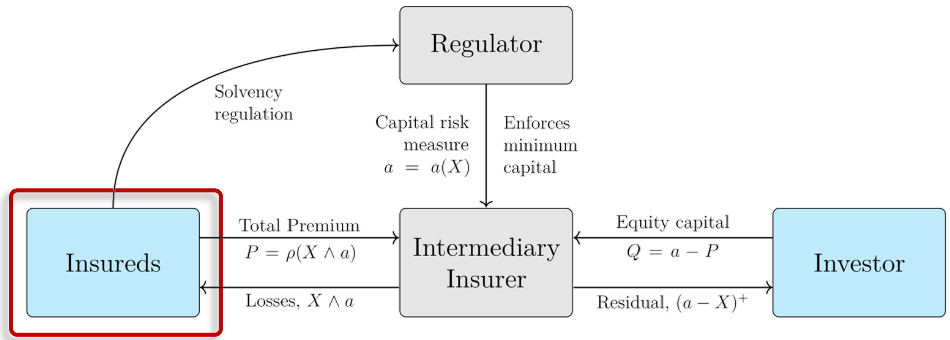
Four actors and market interactions



- Standard simplifying assumptions: no expenses, no investment income
- One-period model
- $X \wedge a = \min(X, a)$

Insured buying behavior

- **Face mandatory / quasi-mandatory insurance requirement**
  - 60% of premium (Aon Benfield, 2015)

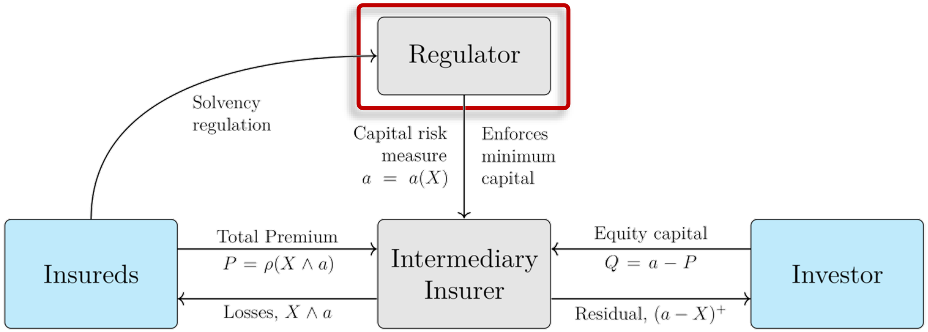


- Mandate is for third-party protection
  - Insureds do not care about insurer solvency provided policy satisfies mandatory requirement, e.g., guarantee funds or judgment proof

- Insureds are pure price buyers, do not see quality differences

Regulator

- **Solvency regulation necessary to ensure mandatory insurance effective**, Cummins (JoF 1988)



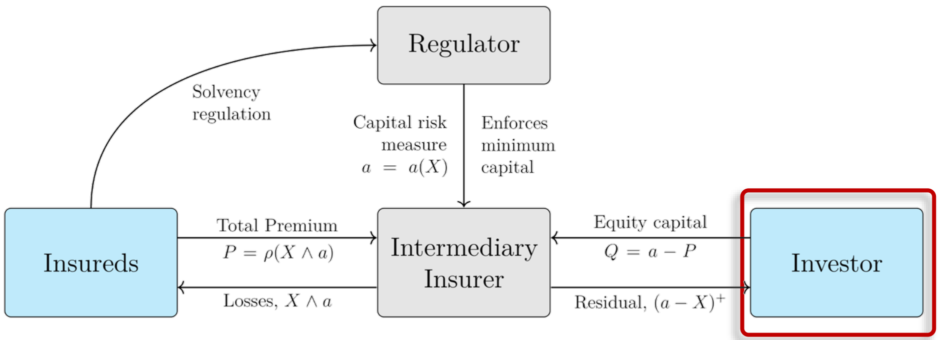
Incorporeal: regulator is a formula

- Regulatory capital standard risk functional $a = a(X) = a(\text{total risk})$
  – **Value at Risk** (VaR) or tail value at risk

- No other regulation beyond capital standard

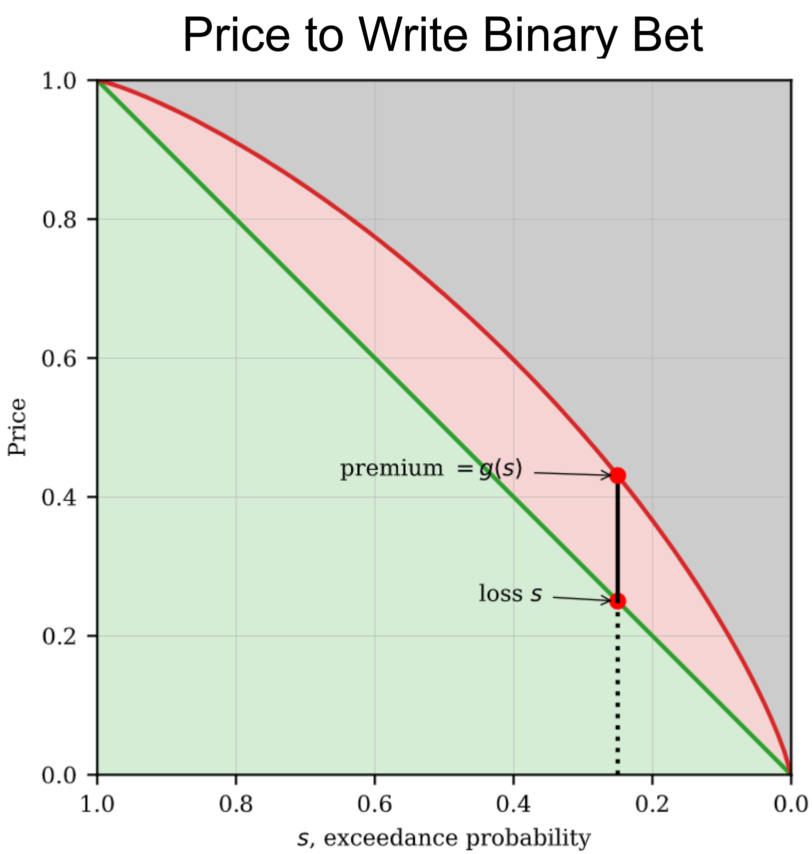
Investor: ultimate risk bearer

- **Charge for risk**, e.g., because **ambiguity averse**, but not necessarily risk averse



- Market price of capital explained by a **distortion risk measure** ρ
  – ρ(X) gives market (ask) price of any loss payout distribution X
  – DRMs are coherent, given by weighted average of TVaRs
  – Law invariant: price of risk only depends on probability of loss

If price of risk only depends on probability of loss...



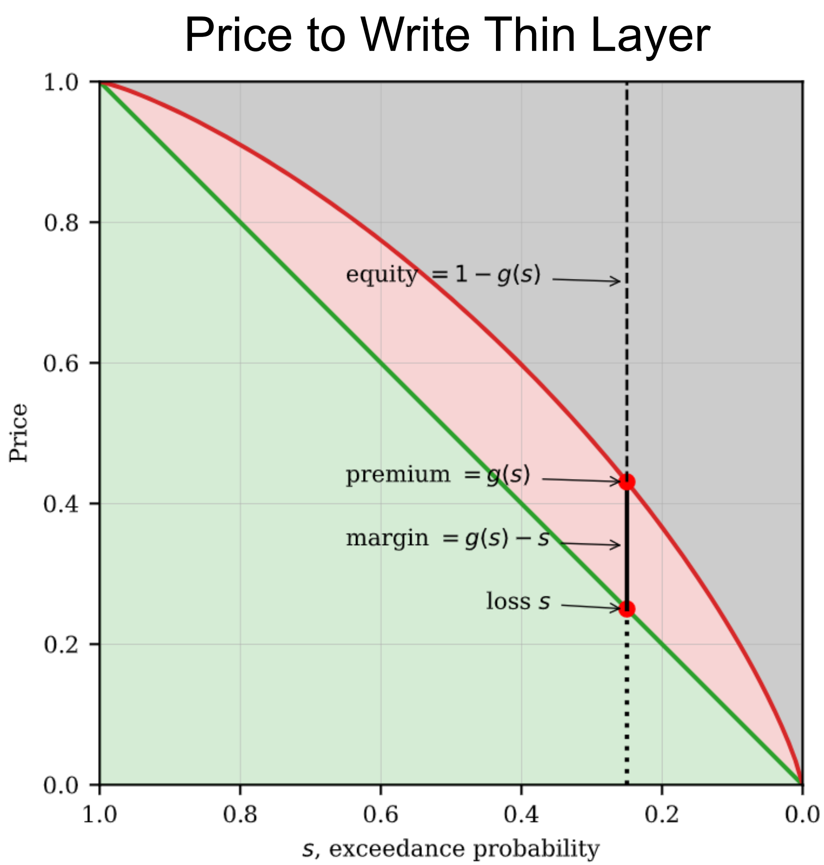
- **Distortion function** g(s) = price to assume risk of paying 1 with probability s, a **thin layer**

- g(0) = 0
- g(1) = 1
- g increasing*
- g concave

- Higher loss = lower probability layers inherently more ambiguous

* Note: x-axis reversed!
Wang Transform, 0.5

Thin layer insurance pricing statistics from distortion function



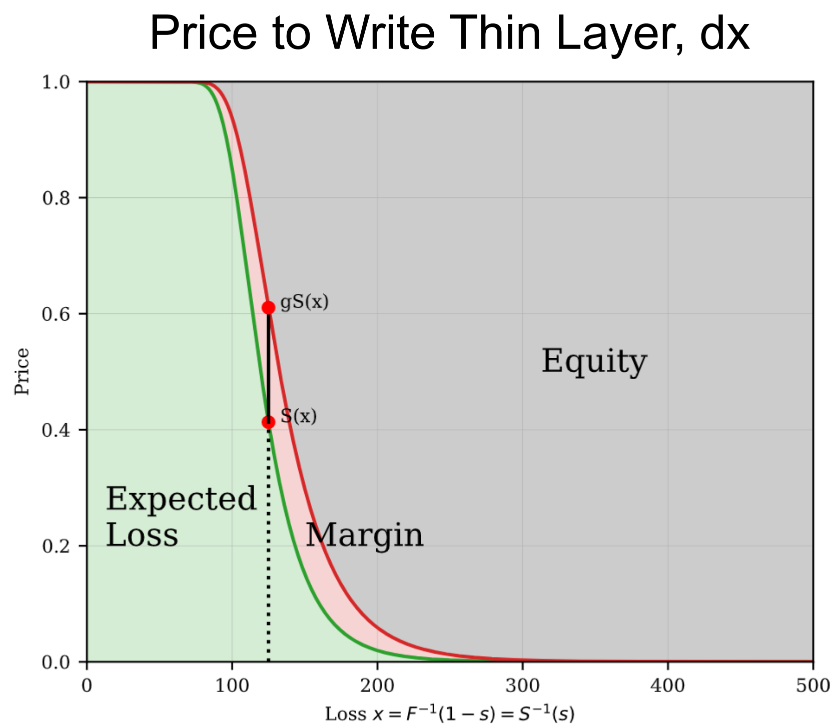
Loss Ratio = $\dfrac{s}{g(s)}$

Premium to surplus leverage = $\dfrac{g(s)}{1-g(s)}$

ROE = $\dfrac{g(s)-s}{1-g(s)}$

Translate from probability of loss to dollars of loss

- Apply inverse distribution function, as per simulation
- Distortion thickens the tail
  - Increases expectation
  - Adds risk margin
- Acts on probabilities not on loss
  - Not a utility adjustment
  - Yarri dual utility
- No objective **events**
  - Events defined implicitly by probability



Limited liability expected loss & pricing implied by a distortion

Expected loss (LEV)

$$E[X \wedge a] = \int_0^a S(x)dx = \int_0^a xf(x)dx + aS(a)$$

distorted probability

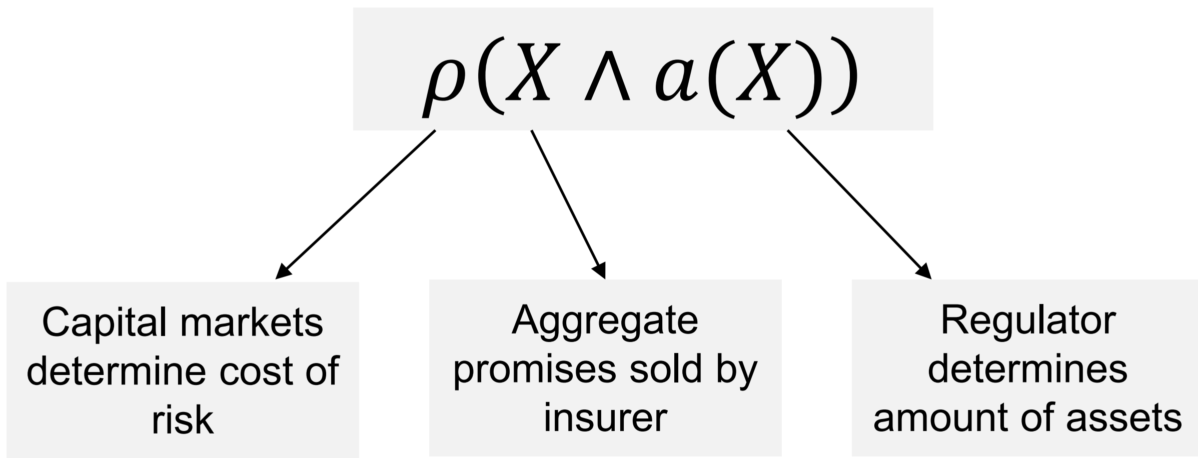
transformed pdf
state price density

Market premium
Distorted expected loss

$$\rho(X \wedge a) = \int_0^a g\big(S(x)\big)dx = \int_0^a xg'\big(S(x)\big)f(x)dx + ag\big(S(a)\big)$$

Average life expectancy: add up number of birthdays (survival) and divide by population

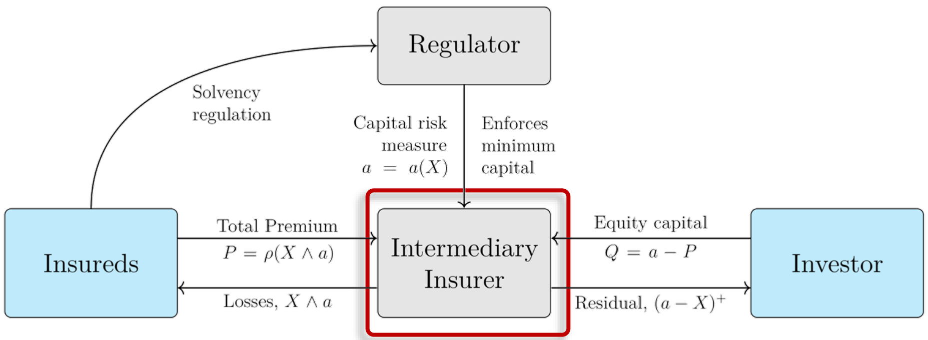
Composite pricing functional



- If functionals ρ and a are monotonic, homogeneous, translation invariant, law invariant then so is composite
- Composite can **fail to be sub-additive** even when ρ and a are both sub-additive because diversification improves coverage quality for $X_0 + X_1$ and hence it costs more

Intermediary insurer

- **Limited liability entity with equal priority in default**



Incorporeal: insurer is a formula

- Operates like a cat bond to minimize **frictional costs** of holding capital
  - No transaction costs, no taxes
  - No management: no principle-agent problems
  - Minimal regulation, no trapped capital
  - Pure exposure to insurance risk, like a sidecars
- Key functions: unambiguous pricing/results and **enable limited liability**

Loss payments: who gets what in default?

- Sold insurance promises

$$X = X_1 + \cdots + X_{\mathrm{n}}$$

- **Equal priority** payment to policy *i* with assets *a*

$$X_i(a) := \begin{cases} X_i & X \le a \\ a\ \ (X_i/X) & X > a \end{cases}$$
$$= X_i \frac{X \wedge a}{X}$$
$$= \frac{X_i}{X} X \wedge a$$

- $X_i(a)$ sum to limited losses, $X \wedge a$

- $\frac{X \wedge a}{X}$ = fixed payment pro rata factor applied to loss from each policy

- $\frac{X_i}{X}$ = variable share of available assets for policy *i* applied to...
- $X \wedge a$ amount of assets available to pay claims

Archetype

- Two policy liabilities (debts)
  - X_0: certain loss, 1000
  - X_1: lognormal, mean 1000, cv 2.0
- Counterparty holds probabilistic reserves, to 90th percentile
  - 1000 for X_0
  - 2272 for X_1
- **Monoline**
  - X_0 no default haircut
  - X_1 has 27% default haircut
- **Pooled**
  - Assets 3272
  - X_1 has access to more assets in event of default, when it captures **more** than 70% (2272/3272) of assets
  - Lowers haircut to 24%
  - 3% transferred from X_0 to X_1
- **Conclusion**
  - Expected value of 970 for X_0, below promised actuarial value

Expected loss and premium allocation by class and layer

$$\text{Expected Loss} = E[X_i(a)] = \int_0^a \underbrace{E\left[\frac{X_i}{X} | X > x\right]}_{\alpha_i(x)} S(x)dx = \int_0^a \alpha_i(x)S(x)dx$$

$$\text{Premium} = \rho(X_i(a)) = \int_0^a \underbrace{E^*\left[\frac{X_i}{X} | X > x\right]}_{\beta_i(x)} g\big(S(x)\big)dx = \int_0^a \beta_i(x)g\big(S(x)\big)dx$$

- X_i/X = variable share of available assets for policy *i*
- All quantities add-up
- No arbitrary choices
- Not marginal cost, not Aumann-Shapley value

Assumptions

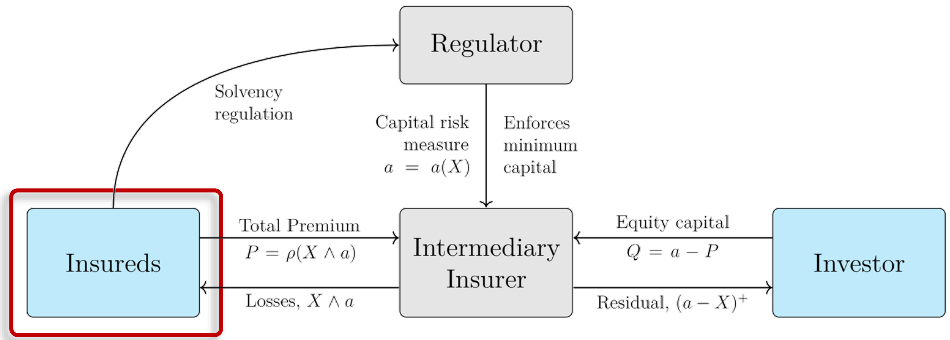
- Price with DRM *g*
- Equal priority in default

Independence of X_i **not** required

Relies on comonotonic additivity of DRM

Insured loss distributions

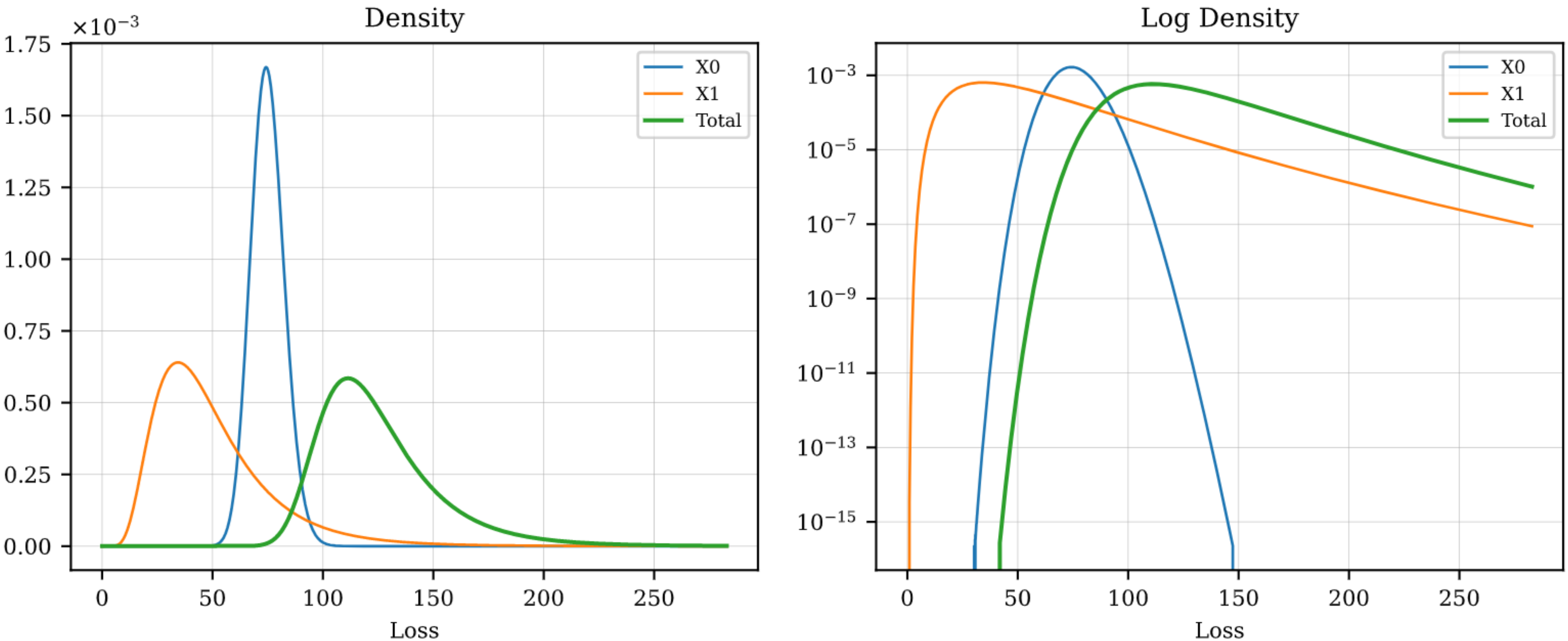
- **Two classes (lines) of insured**
  - X_0 thin-tailed class: high frequency, low severity; **Illinois personal auto**
  - X_1 thick-tailed class: catastrophe exposed; **Florida home**



- Risk is a characteristic of **class** and not the individual insured

- Homogeneous loss model: distribution scales, no shape change
  - Results for a sub-pool of a class are proportional to the results for whole class, i.e., model loss ratio, Myers Read and GBM models are homogenous
  - Mildenhall (Risks 2017)

Example: Thin- and Thick-tailed two-class model



- Classes independent, convenience only
- X_0 thin class, EL 75, CV 10%, gamma distribution, comparable to personal auto
- X_1 thick class, EL 50, CV 53%, lognormal distribution, cat-exposed property
- Portfolio CV 22%
- Initially, expensive pricing, weak capital standard

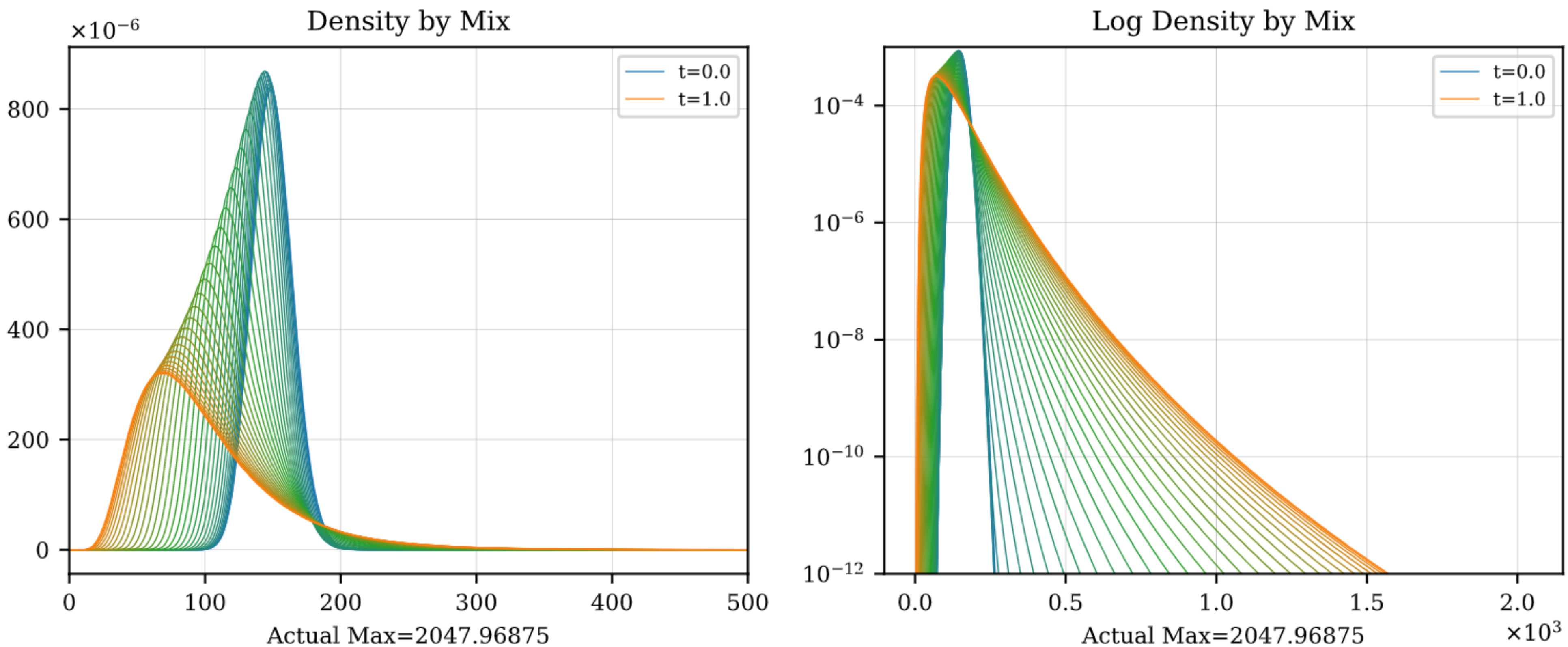
How will risks pool?

- Pools with the same class mix (e.g., monoline) can merge by homogeneity
- Pricing varies with mix: only one multiline pool (cheapest)
- There are only three possible market structures
  - Full pooling: one insurer
  - Two monoline insurers
  - One multiline pool insurer and one monoline insurer

- **Market defined by proportion *t* of risk class 1 in the pool, $0 \le t \le 1$,** and

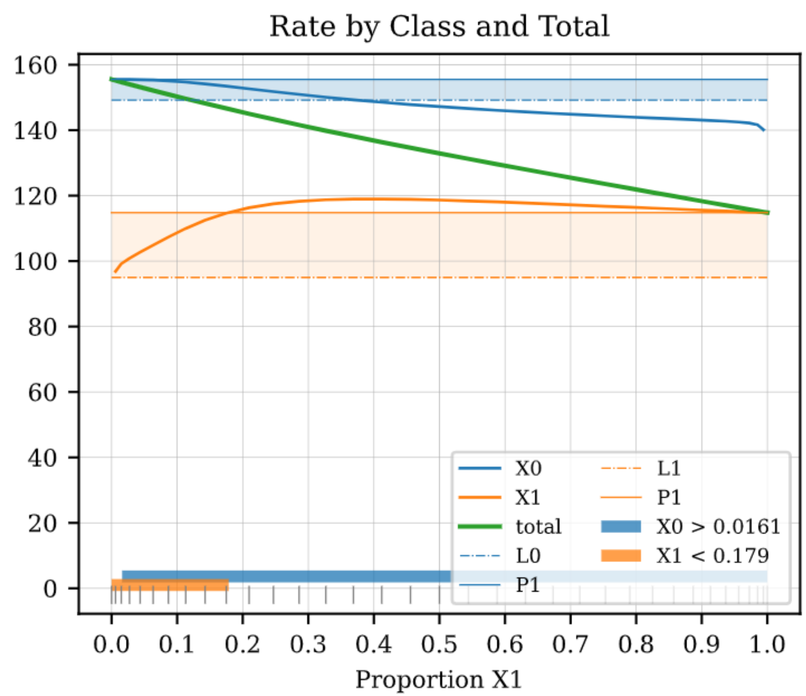
$t = 0, 1$	two monoline pools
$t = 0.5$	full pooling
$0 < t < 0.5$	class 0 fully pooled, class 1 split between pool and monoline
$0.5 < t < 1$	class 1 fully pooled, class 0 split between pool and monoline

Total loss density, by portfolio mix $0 \le t \le 1$



- Pool outcome is $X_t = (1 - t)X_0 + tX_1$
- Computations performed for 35 values of t
- Graphs show how shape of aggregate portfolio transitions from X_0 to X_1

Premium rates by class

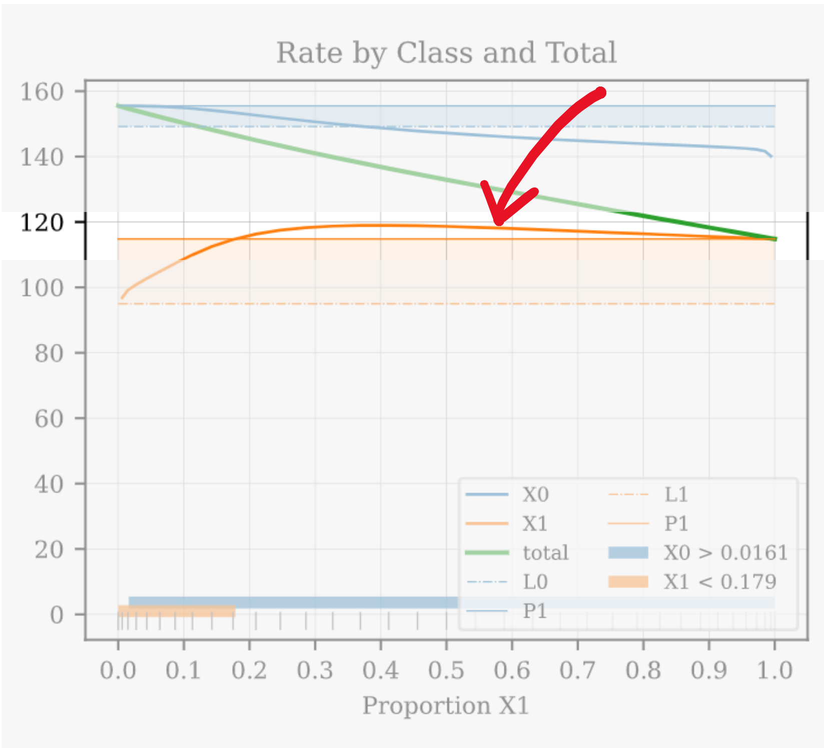


Assumptions

- Wang hazard rho with 0.5 parameter
- Capital standard: 90% value at risk
- Premium rate = allocated premium / proportion of class, is comparable with monoline premium

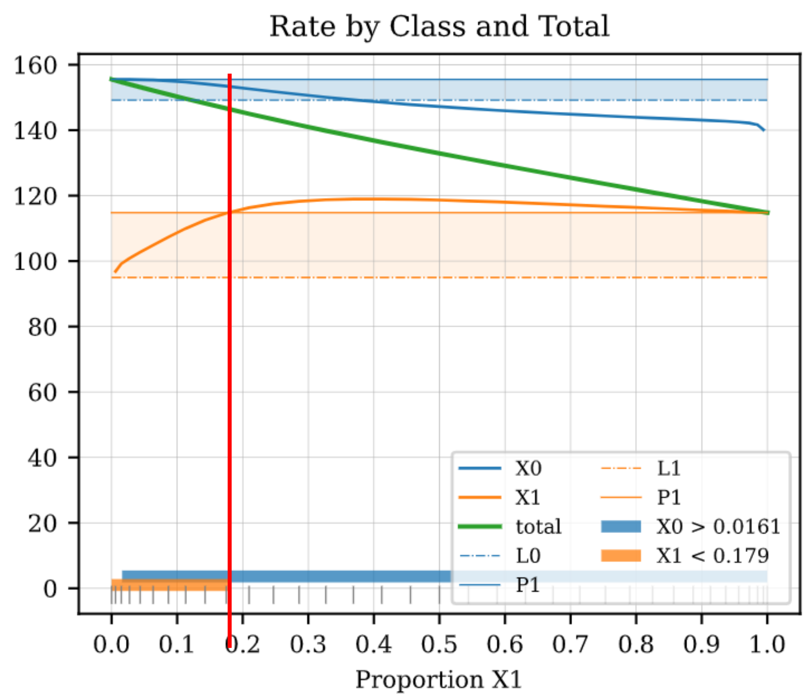
- *t*, the proportion of X_1, on x-axis
- Lines show **rate** for each class
  - **Blue X_0 low, orange X_1 high risk**
  - Green: blended pool rate
- Shaded bands at top show range from monoline loss cost and premium for each class
- Expected unlimited loss, before insurer default X_0 = 150, X_1 = 100; slightly less with limited capital
- Expensive pricing, weak capital standard

Limited liability causes rate to bow up above monoline rate



- Pooling risky debt with certain debt benefits risky debt in default
- Benefit compensated through higher a priori premium
- Pool offers better coverage to riskier insureds = costs more
- Cost to provide insurance even when no benefit received, e.g., basis risk

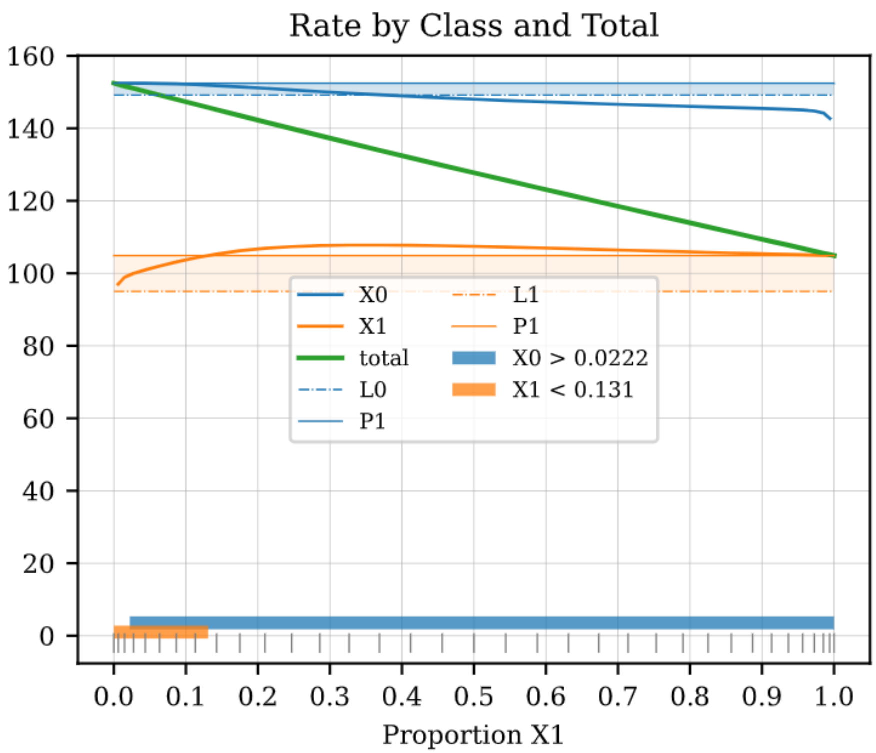
Partial pooling equilibrium solution



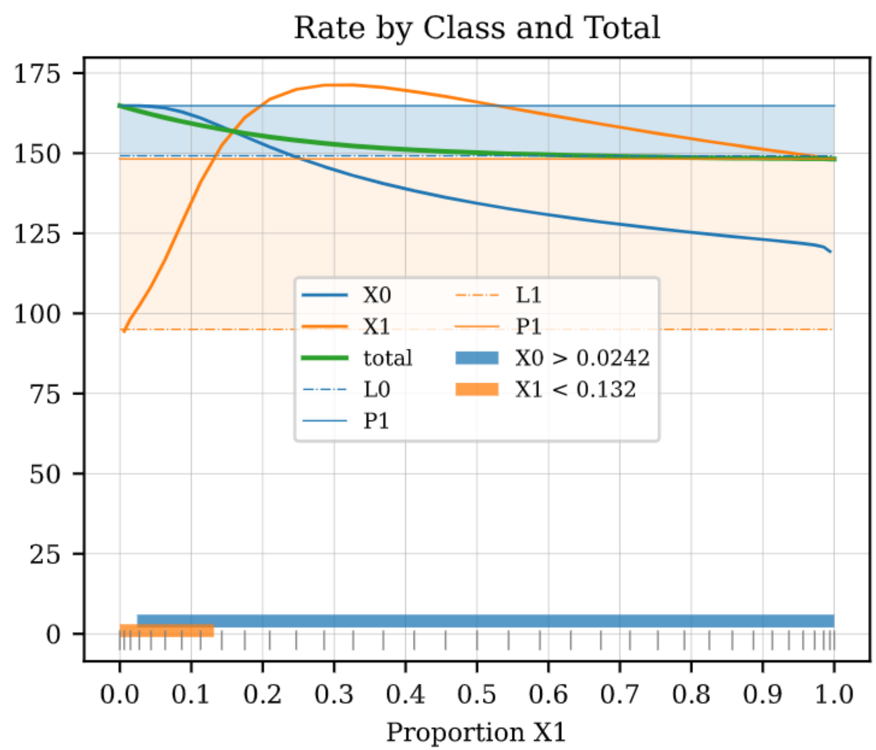
Hence Florida home-owners not fully pooled

- **Equilibrium solution, t = 0.179**
  - X_0 and 22% of X_1 are pooled; remaining 78% of X_1 written monoline
- Why?
  - *t* > 0.179: X_1 rate greater than monoline...X_1 will not pool
  - *t* < 0.179: X_1 insureds in pool get below monoline rate, with remainder monoline
  - Remainder will offer to pool with X_0 at slightly higher rate until equilibrium reached at *t* = 0.179
  - **X_1 pays monoline rate and X_0 captures all diversification benefit**

Sensitivity to cost of capital

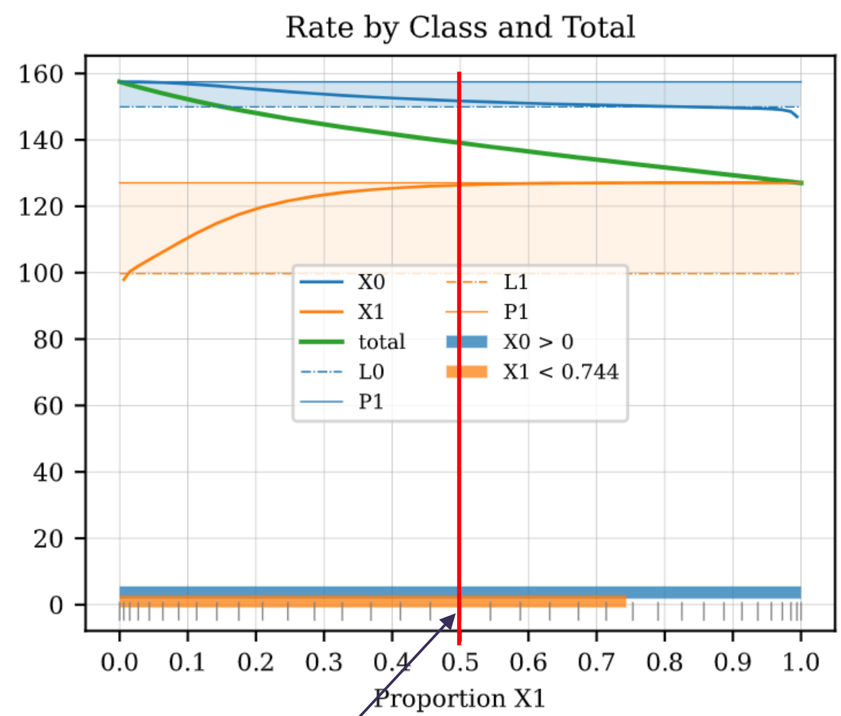


Wang 0.25 parameter



Wang 1.5 parameter

Stricter capital standard leads to full pooling outcome



- Feasible region overlap includes 50/50 pool
- X_1 premium ~ 128 vs. 118 at p=0.9

- 99.5% VaR capital standard (Solvency II level), base Wang 0.5 cost of capital
- **When *t* = 0.5 is feasible for both lines, it is the equilibrium solution**
  - If $t \neq 0.5$, some insureds are forced into monoline rate
  - Monoline insureds offer to pool at more advantageous rate
  - $t \neq 0.5$ pool unravels
- At *t* = 0.5, all insureds pay lower multiline rate and no rational action can cause pool to unravel
- DemoTech in FL offers **weaker** standard

Conclusions

- Pooling solution determined by subtle interaction between
  - Relative tail thickness of X_0 and X_1
  - Strength of capital standard
  - Cost of capital
- Full pooling is more likely with
  - Balanced tail thickness
  - Stronger capital standard
- Impact of cost of capital indeterminate
- Diversification benefit of pooling is eroded by economic transfers caused by limited liability, especially with weak capital standard

Appendix

Audit statistics and pricing summary

	X0	X1	total
Mean	75	50	125
CV	0.1	0.53294	0.221459
Skew	0.2	1.75019	1.56504
EmpMean	74.9844	49.9844	124.969
EmpCV	0.100021	0.533107	0.221514
EmpSkew	0.2	1.75019	1.56504
EmpKurt	0.0599998	5.89843	5.06461
P90.0	84.75	83.7188	159.938
P95.0	87.7188	100.406	176.625
P99.9999	116.188	475.188	550.812

	99.5% VaR			90.0% VaR		
	X_0	X_1	**Total**	X_0	X_1	**Total**
Item						
1. Allocated assets	110.602388	125.428862	236.031250	84.666785	75.270715	159.937500
2. Market value liability	75.856673	63.192198	139.048871	73.600431	59.324711	132.925142
3. Expected incurred loss	74.945567	49.875561	124.821128	74.052542	48.430830	122.483372
4. Margin	0.911106	13.316637	14.227743	-0.452111	10.893881	10.441769
5. Loss ratio	0.987989	0.789268	0.897678	1.006143	0.816369	0.921446
6. Allocated equity	34.745715	62.236664	96.982379	11.066354	15.946004	27.012358
7. Cost of allocated equity	0.026222	0.213968	0.146704	-0.040855	0.683173	0.386555
8. Premium to surplus ratio	2.183195	1.015353	1.433754	6.650829	3.720350	4.920901

- Example produced using aggregate Python package
  https://github.com/mynl/aggregate
  https://aggregate.readthedocs.io/
- `pip install aggregate`
- `aggregate` program for t=0.50 portfolio

```
port MIX_thin_thick
      agg X0 1 claim sev gamma 75.0 cv 0.1 fixed
      agg X1 1 claim sev lognorm 50.0 cv 0.5329 fixed
```

- Pricing results using 99.5% VaR and 90.0% capital and Wang 0.5 distortion for t=0.50 portfolio
- Market value liability = premium
- Note: by class rates shown in graphs are twice (divide by 0.5) the amounts shown here

Expected loss and premium allocation by class and layer

Expected Loss = $E[X_i(a)] = \int_0^a \underbrace{E\left[\frac{X_i}{X} | X > x\right]}_{\alpha_i(x)} S(x)dx = \int_0^a \alpha_i(x) S(x) dx$

Premium = $\rho(X_i(a)) = \int_0^a \underbrace{E^*\left[\frac{X_i}{X} | X > x\right]}_{\beta_i(x)} g\big(S(x)\big)dx = \int_0^a \beta_i(x) g\big(S(x)\big)dx$

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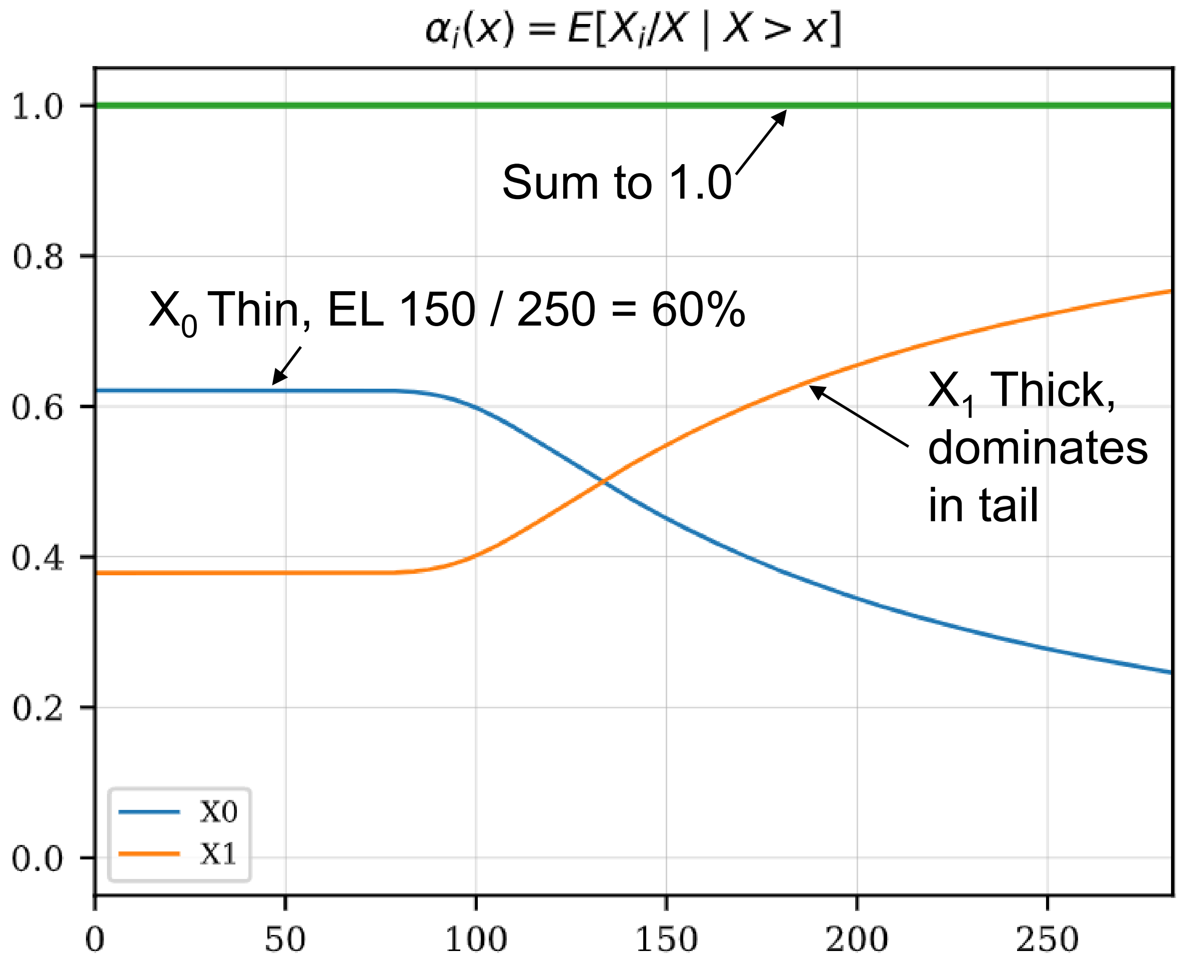
Assumptions

- Price with DRM *g*
- Equal priority in default

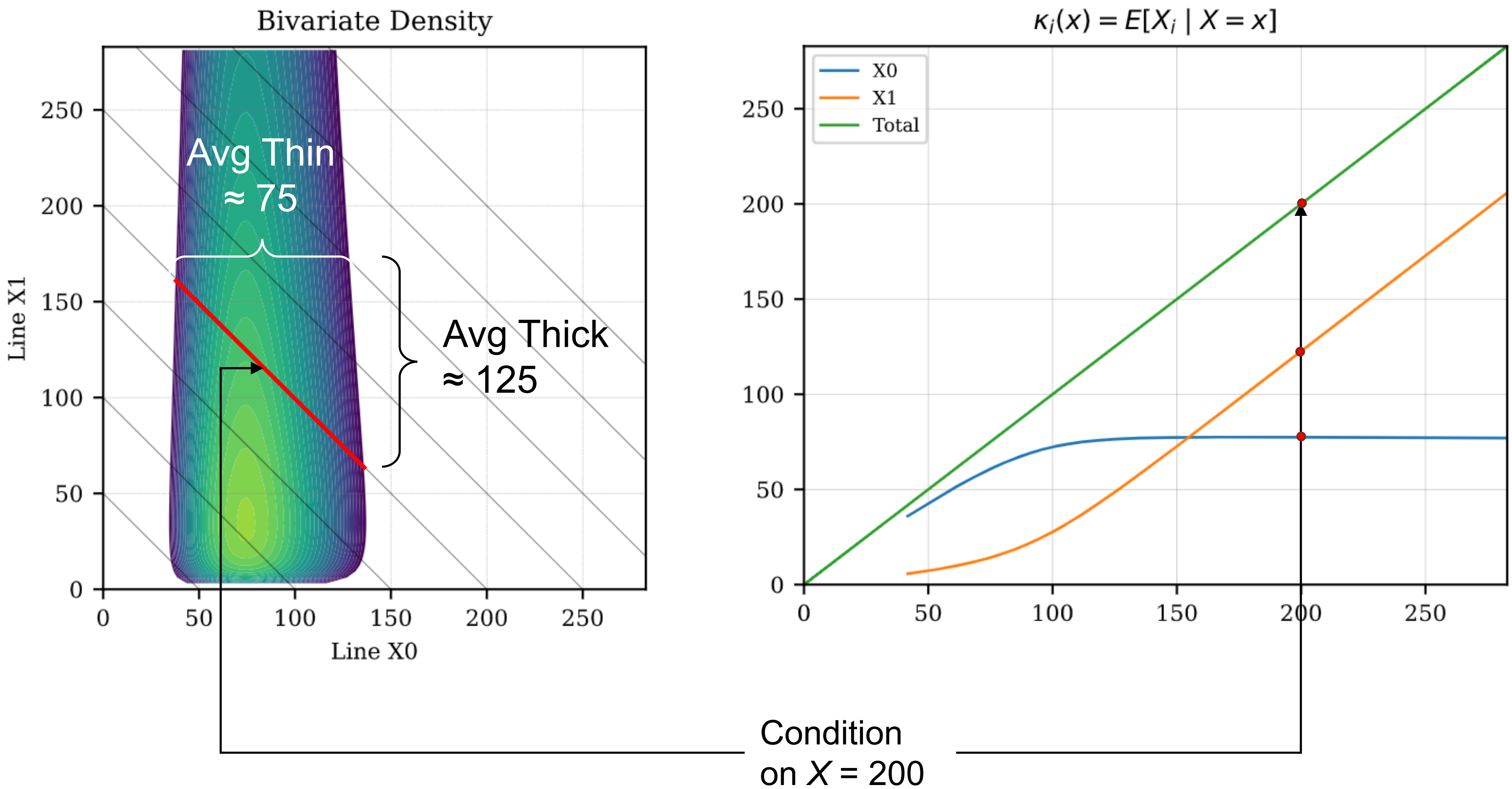
Independence of X_i **not** required

alpha function: proportion of expected loss by layer



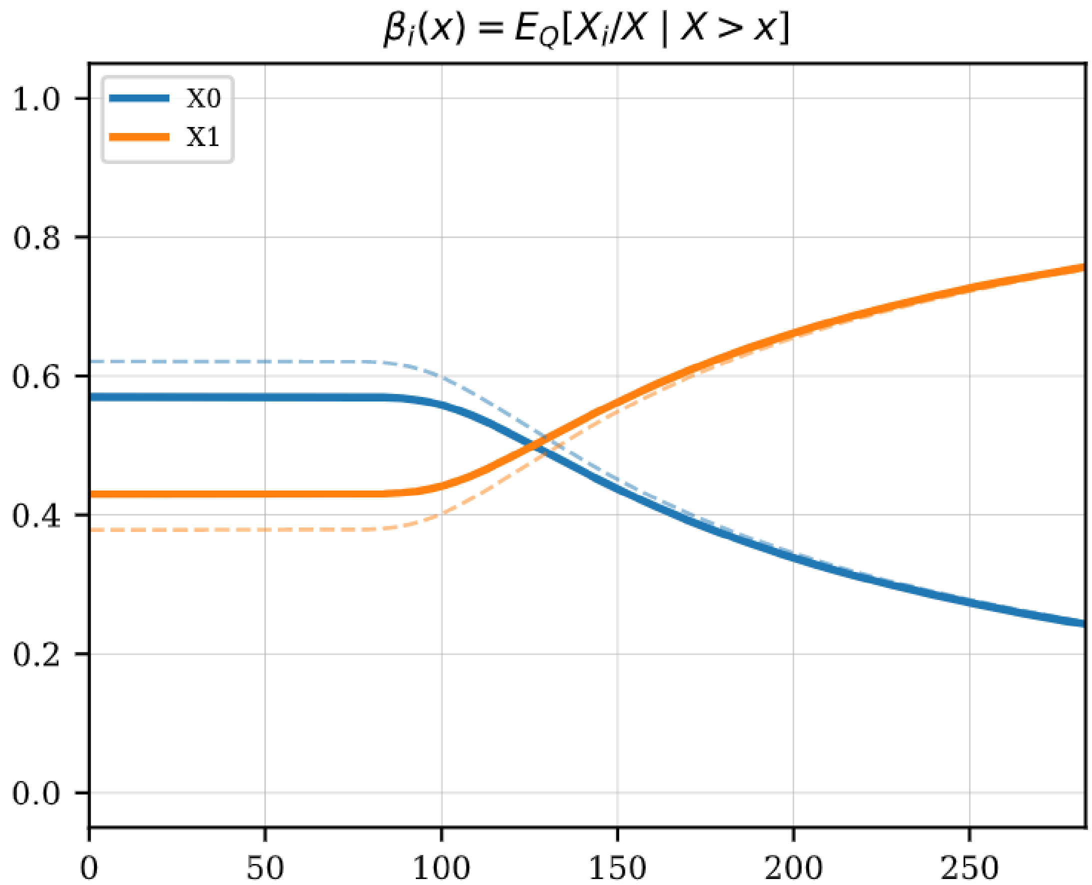
$$\alpha_i(x)S(x) = \int_x^\infty \frac{E[X_i \mid X = t]}{t} f_X(t)dt$$

E[X_i | X=x]: building block function for alpha and beta



beta function: proportion of premium by layer

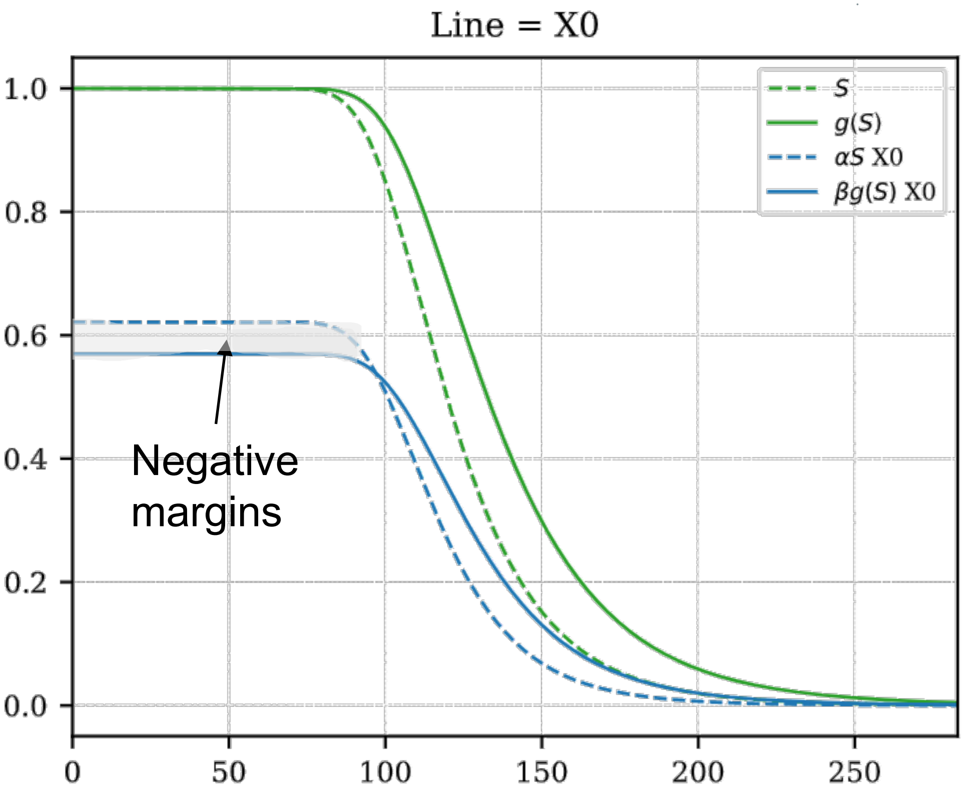
- $\beta_i(x)$, solid line, is a risk adjusted version of $\alpha_i(x)$, dashed, putting more weight on right tail

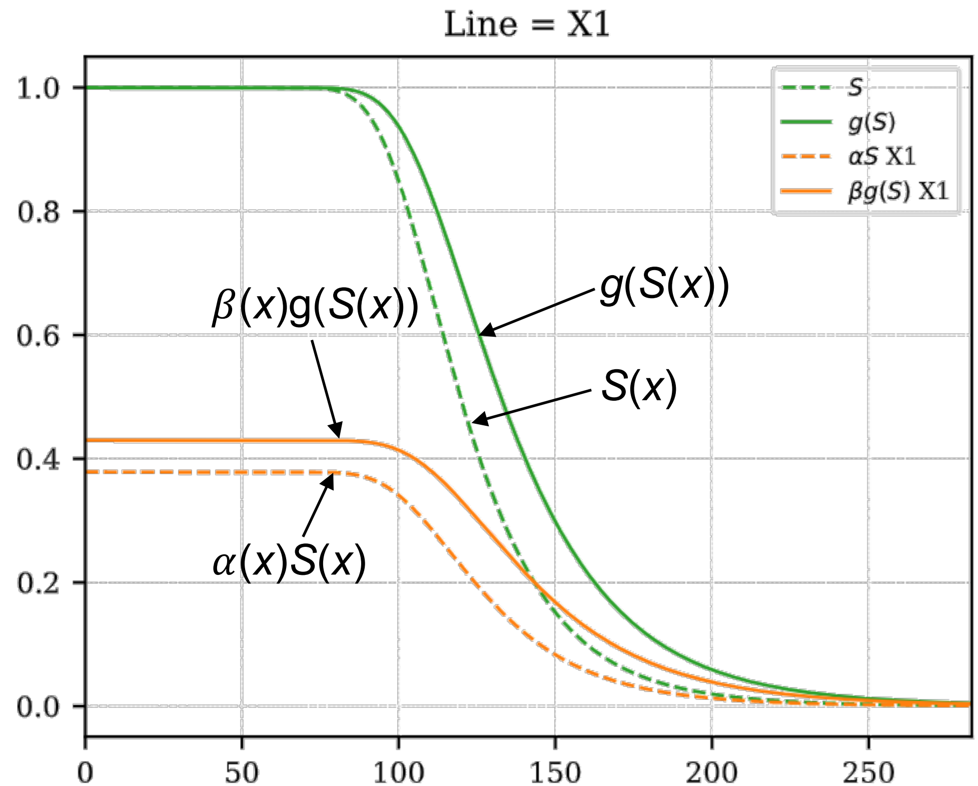


When $\alpha_i(x)$ **inc**reases $\beta_i(x)$ is **above** $\alpha_i(x)$, positive margins = **Thick orange** (solid above dashed)

When $\alpha_i(x)$ **dec**reases $\beta_i(x)$ is **below** $\alpha_i(x)$, negative margins for some layers = **Thin blue**

Margins by asset layer, by class

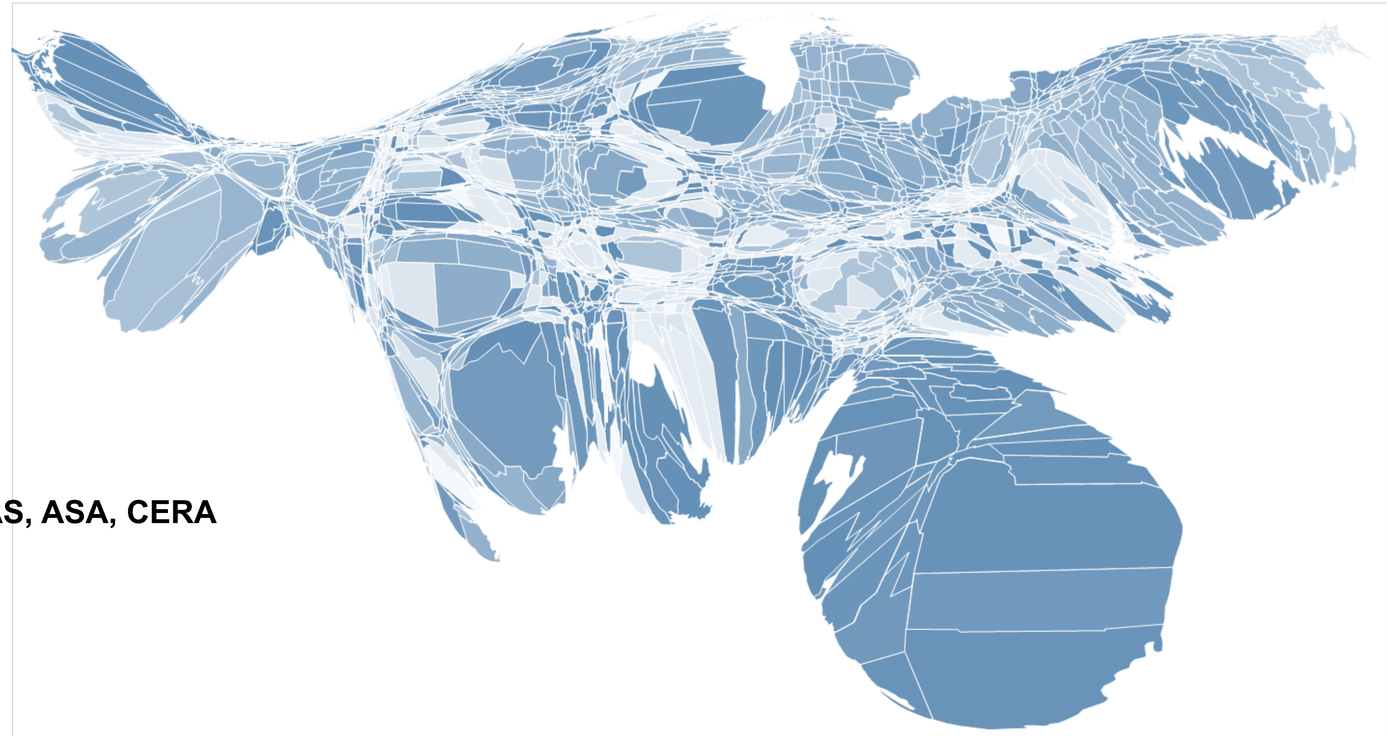




- Thin...$\alpha_i(x)$ **dec**reases...$\beta_i(x)$ **below** $\alpha_i(x)$
- $\beta_i(x)g(S(x))$ may be **below** $\alpha_i(x)S(x)$
- Possible negative margins for low layers
- Eventual cumulative margin positive

- Thick...$\alpha_i(x)$ **inc**reases...$\beta_i(x)$ **above** $\alpha_i(x)$
- $\beta_i(x)g(S(x))$ **above** $\alpha_i(x)S(x)$ since $g(S)>S$
- Positive margins at all layers of capital

Contact Information



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Graphic details: County size scaled to AAL estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm